PROBABILITY

EMPIRICAL:
relationship or expectation is found by experiment or use of historical data

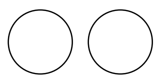
THEORETICAL
expectation is found by use of logic, symmetry or listing outcomes

SUBJECTIVE
based on belief or judgement of what will happen

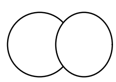
Some basics…….

EXCLUSIVE AND NON-EXCLUSIVE EVENTS

Two events are mutually exclusive if they are not related



Two events are not exclusive or joint if they can occur together



Examples:
- throwing a 1 or 6
- picking a Heart or a Picture card

RULES OF ADDITION

If events are exclusive:

Prob (A or B) = P(A u B)
= P(A) + P(B)

P(1 or 6) = P (K or Q) =

If events are not exclusive:

P(A u B) = P(A) + P(B) - P(A n B)

P(Heart or Queen) =

P(Spade or Picture) =

A picture or listing outcomes helps

Investigate throwing two dice and adding the totals:

How many possible outcomes are there?

Find the probabilities of throwing:

- a double
- a total of 10
- a double or a total of 10
- at least one 6

6	.	.	.	.	.	.
5	.	.	.	.	.	.
4	.	.	.	.	.	.
3	.	.	.	.	.	.
2	.	.	.	.	.	.
1	.	.	.	.	.	.
	1	2	3	4	5	6

INDEPENDENT, DEPENDENT AND CONDITIONAL PROBABILITIES

Two events are INDEPENDENT if the occurrence of one is not affected by the occurrence of the other one

example: throwing a die

If an event depends on or is affected by what has happened before then the events are DEPENDENT or the second event is CONDITIONAL on the first

example: passing an exam second go

MULTIPLICATIVE RULES

If events are **independent**

$$P(A \cap B) = P(A) \cdot P(B)$$

e.g: P(two sixes) = P(6) . P(6)

B/A means B once A has happened

If events are **dependent**

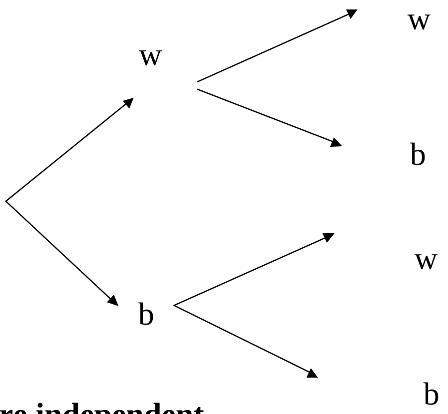
$$P(B / A) = \frac{P(A \cap B)}{P(A)}$$

e.g: $P(<4 \,/\, \text{Heart}) = \frac{P(<4 \text{ and Heart})}{P(\text{Heart})}$

TREE DIAGRAMS

3 white and 7 blue balls in a bag

1. If a ball is selected and then replaced…



Events are independent

TREE DIAGRAMS

3 white and 7 blue balls in a bag

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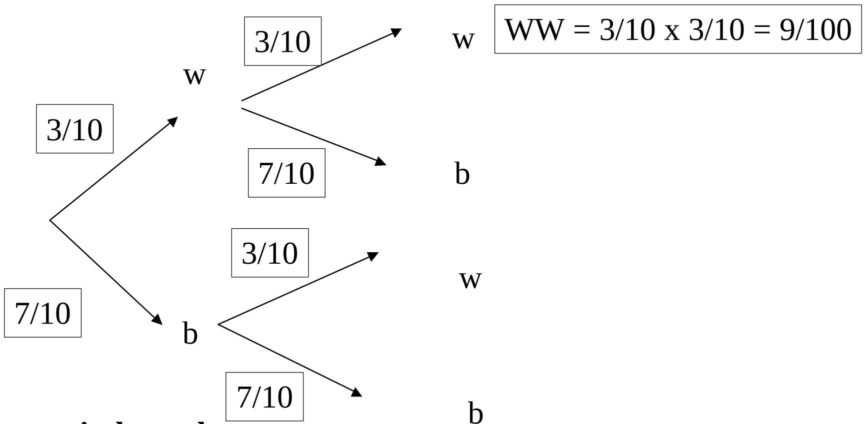
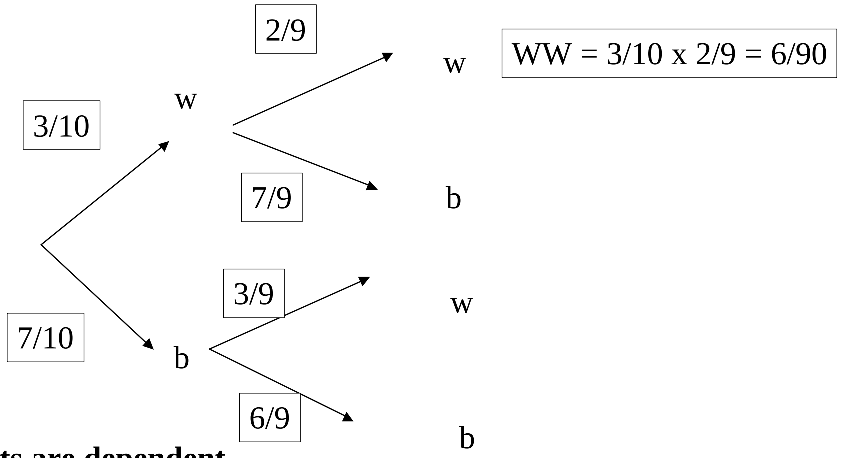


Events are independent

2. If a ball is selected but not replaced…



Events are dependent

The results of a survey of 250 customers at a jeans store can help us in marketing:

Age	Male	Female	totals
< 30	100	75	175
30+	50	25	75
totals	150	100	250

JOINT PROBABILITIES involve two classifications

P(M and 30+) = 50/250 = 0.2

MARGINAL PROBABILITIES are found on edge of table

P(F) = 100/250 = 0.4

CONDITIONAL PROBABILITIES
a probability once another condition has occurred

P(F / <30) =   75/175 = 0.429

Age	Male	Female	totals
< 30	100	75	175
30+	50	25	75
totals	150	100	250

Condition

<30

Find the percentage or probability give info about the customers and underlying trends:

age or gender profile e.g. P(30+)   P(F)

conditional probabilities look for a pattern:
age versus gender
e.g. P(30+/ F) and P(30+/M)
or P(F/<30) and P(M/<30)

Age	Male	Female	totals
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totals	150	100	250

BAYESIAN PROBABILITY

Thomas Bayes - Minister 1702 - 61

- He found a way to estimated the probability of an event that had already happened by using information from a sample

PRIOR PROBABILITY
based on historical data

POSTERIOR PROBABILITY
uses historical data and new information from surveys, testing etc.

Prob of A after B has happened

Prob A * conditional prob of B after A has happened

The formula is

$$P(A / B) = \frac{P(A).\ P(B / A)}{P(A).\ P(B / A) + P(A').\ P(B / A')}$$

Total prob of B

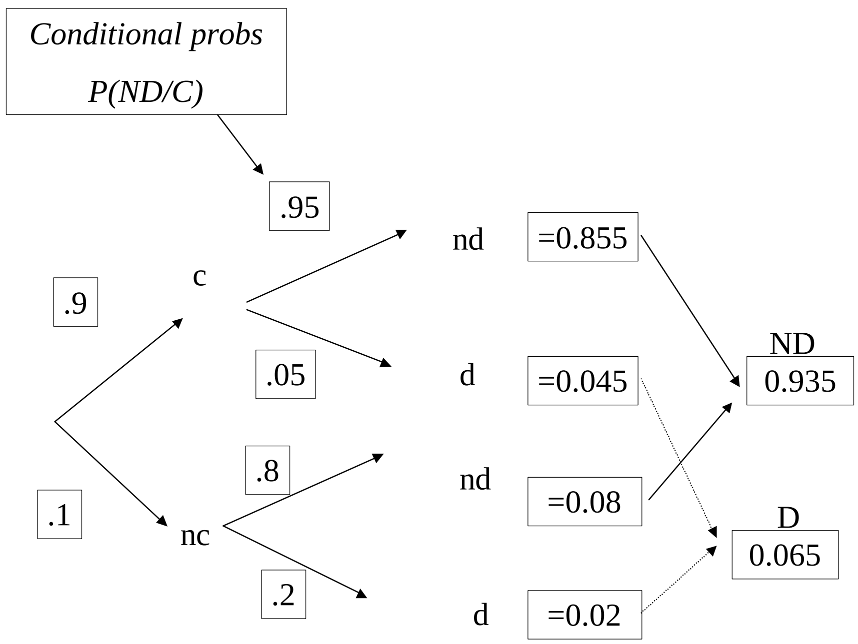
Do not panic!

The formula you find in text books look complicated but using a tree diagram is straightforward and gives the same answer!

A Manufacturing process needs to meet specifications. When it does the process is **In Control.**

- If it is in control the proportion of defectives is 0.05
- If it is out of control the proportion of defectives is 0.20
- Historical data suggests it is in control 90% of the time

We can up-date this historical info using Bayes…..



Sampling:

If you pick out a non-defective item, the probability of the process being in control is:

P(C/ND) = 0.855/0.935 = 0.914 i.e. 91.4%

If you pick out a defective item, the probability of the process being in control is:

P(C/D) = 0.045/0.068 = 0.662 i.e. 66.2%

The Prior probability of 0.9 is up-dated according to sample state

Analyse the following data using different probabilities.

What is the relationship between age, dress and buying behaviour?

A probability case study - based on actual data in USA

- 12 Up - market fashion stores selling women clothing.
- These attract many window shoppers and tourists.
- It would be useful if staff could identify serious buyers.
- The M.R. thinks that buying pattern is affected by age and dress of the shoppers.

- Data is collected recording the behaviour of a random selection of shoppers in one store.

Data One: Female buyers		under 40	40 plus
well dressed	buyer	2	8
	non-buyer	16	14
casually dressed	buyer	34	6
	non-buyer	50	70

Sub-totals are useful….

Data One: Female buyers		under 40	40 plus	total
well dressed	buyer	**2**	**8**	10
	non-buyer	**16**	**14**	30
		18	22	40
casually dressed	buyer	**34**	**6**	40
	non-buyer	**50**	**70**	120
		84	76	160
		102	***98***	***200***

The following information comes from the same store, but for male buyers.

Analyse the data using a tree diagram and Bayes.

Who should the shop assistant target?

Male buyers

- form 1/3 rd of customers
- 6 out of 10 made a purchase
- of those who made a purchase 2 out of 10 wore suits
- of those who didn't make a purchase 9 out of 10 were not in a suit